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UPPER EVALUATION OF POWER OF SURFACE FORCES WITH DEFORMATION OF A MEDIUM WITH A LIMITED INTENSITY OF TANGENTIAL STRESSES

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UDC 539.3

We consider a medium in which the intensity of the tangential stresses cannot exceed a given value. In other respects, the medium is arbitrary: The connection between the stresses and the deformations can be arbitrary; specifically, the deformation can be accompanied by a breakdown of the continuity (fracture). With the deformation of such a medium, the power of the forces at that part of the surface where the velocities are given can be evaluated from above [1]. In the present article a more general evaluation is proposed, based on the use of the kinematically possible field of the velocity and a model of an inhomogeneous viscous incompressible liquid. If the viscosity coefficient is determined from the condition of a minimum of the evaluation, it coincides with the known value [1]. The use of the proposed evaluation makes it possible to obtain simple evaluations of the power of the surfaces and to calculate by successive approximations the minimal evaluation in a given class of kinematically possible velocities.

§ 1. Upper Evaluation of Power of Surface Forces

Let σ_{ij}^* be any stresses, in the region Ω, satisfying the equilibrium equations

$$\sigma_{ij,j}^* + f_i = 0,\ i = 1, 2, 3, \tag{1.1}$$

the inequality

$$\sigma_{ij}'^*\sigma_{ij}'^* \leqslant 2\tau^2,\ \sigma_{ij}'^* = \sigma_{ij}^* - \delta_{ij}\sigma^*,\ \sigma^* = \frac{1}{3}\delta_{ij}\sigma_{ij}^*, \tag{1.2}$$

and, in the part S_σ of the boundary S of the region Ω, the conditions

$$\sigma_{ij}^*\nu_j = p_i,\ i = 1, 2, 3. \tag{1.3}$$

In (1.1)-(1.3) and in what follows a Cartesian system of coordinates is used; f_i, p_i are given functions; τ is a constant; ν_i are the components of an external unit normal to the surface S.

Let u_i^* be the components of the vector of the velocity, given at S_u; $S_u = S - S_\sigma$; u_i is some kinematically possible field of the velocities, i.e., a field of the velocities satisfying the condition of incompressibility in Ω

$$\delta_{ij}u_{i,j} = 0 \tag{1.4}$$

and the condition at S_u

$$(u_i - u_i^*)\nu_i = 0. \tag{1.5}$$

The velocities u_i can have tangential discontinuities at some surfaces S_k, $k = 1, 2, \ldots, m$.

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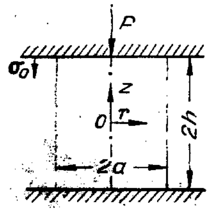

Fig. 1

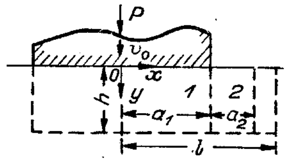

Fig. 2

We denote

$$\sigma'_{ij} = \mu e_{ij},\ e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

where μ is some positive function of the coordinates. From (1.2) we find

$$\sigma'^{*}_{ij}\sigma'_{ij} \leqslant \tau^2 + \frac{1}{2}\sigma'_{ij}\sigma'_{ij},$$

and, consequently,

$$\sigma'^{*}_{ij} e_{ij} \leqslant \frac{1}{\mu}\left(\tau^2 + \frac{1}{2}\mu^2 e_{ij}e_{ij}\right). \tag{1.6}$$

From (1.1)-(1.5) we find

$$\int_{\Omega} \sigma'^{*}_{ij} e_{ij} d\Omega \geqslant \int_{S_u} \sigma^{*}_{ij}\nu_j u_i dS + \int_{S_\sigma} p_i u_i dS + \int_{\Omega} f_i u_i d\Omega - \tau \sum_{k=1}^{m} \int_{S_k} [u]_k\, dS,$$
$$\int_{S_u} \sigma^{*}_{ij}\nu_j u^{*}_i dS \leqslant \int_{S_u} \sigma^{*}_{ij}\nu_j u_i dS + \tau \int_{S_u} [u]\, dS, \tag{1.7}$$

where $|u|_k$ is the modulus of the tangential discontinuity of the velocities u_i at S_k; [u] is the modulus of the difference in the tangential components of the velocities u_i^* and u_i at S_u.

From (1.6), (1.7), it follows that

$$\int_{S_u} \sigma^{*}_{ij}\nu_j u^{*}_i dS \leqslant \int_{\Omega} \frac{1}{\mu}\left(\tau^2 + \frac{1}{2}\mu^2 e_{ij}e_{ij}\right) d\Omega +$$
$$+ \tau\left(\sum_{k=1}^{m} \int_{S_k} [u]_k dS + \int_{S_u} [u]\, dS\right) - \int_{\Omega} f_i u_i d\Omega - \int_{S_\sigma} p_i u_i dS. \tag{1.8}$$

The inequality (1.8) contains an arbitrary positive function μ. If it is defined from the condition of a minimum of the right-hand part of (1.8), then

$$\mu = 2\tau/H,\ H^2 = 2e_{ij}e_{ij} \tag{1.9}$$

and inequality (1.8) goes over into the well-known evaluation [1] of the power of the external forces at S_u for a medium with a limited intensity of the tangential stresses (1.2).

Let the kinematically possible velocities be given as functions of the coordinates and the parameters c_k, k = 1, 2, ..., N. We denote the right-hand part of (1.8) by $G(\mu, c_k)$. To find the values of c_k for which $G(\mu, c_k)$ is maximal, in the case (1.9) a system of equations nonlinear with respect to c_k must be solved. A solution of this system can be obtained by successive approximations. Obviously,

$$G(\mu_n c_k^n) \geqslant \min_{\mu} G(\mu, c_k^n) = G(\mu_{n+1}, c_k^n) \geqslant \min_{c_k} G(\mu_{n+1}, c_k) = G(\mu_{n+1}, c_k^{n+1}),$$

where μ_n, c_k^n, n = 0, 1, ..., is the sequence of the approximate functions μ and the sought values of the parameters c_k. The system of equations of each of the approximations is linear with respect to those parameters c_k on which the kinematically possible field of the velocities and the discontinuities of $[u]_k$, [u] depend linearly.

Using (1.8), it is simple to obtain calculated evaluations, assuming, for example, that μ is a constant or piecewise-constant function in Ω.

§ 2. Axial Compression of a Round Cylinder by Flat Slabs

Let a round cylinder with a diameter of $2a$ and a height of 2h (Fig. 1) be compressed by absolutely hard slabs, so rough that the radial displacements of particles of the cylinder at the contact planes are equal to zero.

Let the kinematically possible axial velocity w be a cubic polynomial in z, whose coefficients do not depend on r; the kinematically possible radial velocity u is linear with respect to r. From the condition of incompressibility and the conditions

$$w|_{z=h} = -w_0,\ w|_{z=0} = \partial u/\partial z|_{z=0} = 0$$

we find

$$w = -\zeta w_0[1 + c(1 - \zeta^2)],\ u = -(1/2)r\,dw/dz,$$
$$e_{ij}e_{ij} = (1/2)[3(dw/dz)^2 + (1/4)r^2(d^2w/dz^2)], \tag{2.1}$$

where $\zeta = z/h$; c is a parameter of the kinematically possible field of the velocities; w_0 is the velocity of the motion of a slab. We calculate the upper evaluation of the force P of the compression of the cylinder, assuming that, in (1.8), in the whole volume of the cylinder, the function μ is equal to exactly the same constant. From (1.8), (2.1), it follows that

$$p \leqslant 1/\mu_* + (1/40)[10 + (8 + 5\lambda^2)c^2]\mu_* + \lambda(1 - 2c)/3\sqrt{3}, \tag{2.2}$$

where

$$p = P/\pi a^2\tau\sqrt{3};\ \mu_* = \mu w_0\sqrt{3}/\tau h,\ \lambda = a/h.$$

Minimizing the right-hand part of (2.2) with respect to c, and then with respect to μ_*, we find

$$p \leqslant [1 - 40\lambda^2/27(8 + 5\lambda^2)]^{1/2} + \lambda/3\sqrt{3}.$$

In the whole range of change in λ, the evaluation found differs only slightly from the results of a calculation of the minimal evaluation corresponding to the kinematic field under consideration [1].

§ 3. The Impression of a System of Flat, Smooth Punches

Let a system of absolutely hard punches with flat bases be pressed with the velocity v_0 into the flat surface of a medium in such a way that the deformation is plane; the deformation is symmetrical with respect to the lines $x = 0$, $x = \pm l$ (Fig. 2).

As a kinematically possible field of the velocities in region 1 we take

$$u = \xi[1 + c(\eta - 1/2)]v_0,\ v = (1 - \eta)[1 + (1/2)c\eta]v_0; \tag{3.1}$$

in region 2

$$u = -(\xi - \alpha_1 - \alpha_2)[1 + c(\eta - 1/2)]v_0/\lambda,\ v = -(1 - \eta)[1 + (1/2)c\eta]v_0/\lambda, \tag{3.2}$$

where $\xi = x/h$; $\eta = y/h$; $\alpha_1 = a_1/h$; $\alpha_2 = a_2/h$; $\lambda = a_2/a_1$; c, h are the parameters of the field of the velocities; v_0 is the velocity of the motion of the punches. We calculate the upper evaluation of the force P of the impression of a punch, assuming that there is no friction at the contact surfaces; $a_1 + a_2 = l$; the function μ in (1.8) is equal to a constant value of μ_1 in region 1 and to a constant value of μ_2 in region 2. From (1.8), (3.1), and (3.2) it follows that

$$2p \leqslant 1/\mu_1^* + \mu_1^*[1 + c^2(1 + \alpha_1^2)/12] + 1/\mu_2^* + \mu_2^*[1 + c^2(1 +$$
$$+ \alpha_2^2)/12] + \frac{1}{2}\left[(\alpha_1 + \alpha_2)\left(1 + \frac{1}{2}c\right) + \left(1 + \frac{1}{6}c\right)(1/\alpha_1 + 1/\alpha_2)\right], \tag{3.3}$$

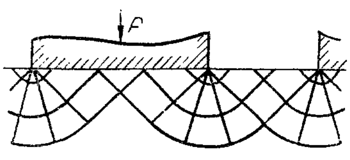

Fig. 3

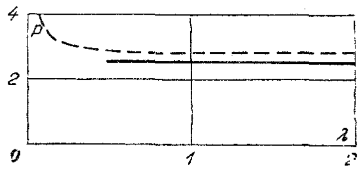

Fig. 4

where

$$p = P/4a_1\tau; \quad \mu_1^* = \mu_1 v_0/\tau h; \quad \mu_2^* = \mu_2 v_0/\tau h\lambda.$$

A solution of the nonlinear system of equations with respect to the values of μ_1^*, μ_2^*, c, h, corresponding to a minimum evaluation of (3.3), can be obtained by successive approximations. We limit ourselves to the first approximation. Determining μ_1^*, μ_2^*, h from a minimum of the evaluation of (3.3) with c = 0, we find

$$p \leqslant 2 + (1+\lambda)/2\sqrt{\lambda}, \quad \mu_1^* = \mu_2^* = 1, \quad h = \sqrt{a_1 a_2}.$$

Determining c from the condition of a minimum of the evaluation of (3.3) with the values found for μ_1^*, μ_2^*, h, we obtain

$$p \leqslant 11/6 + (1+\lambda)/2\sqrt{\lambda}, \; c = -2\sqrt{\lambda}/(1+\lambda). \tag{3.4}$$

In the case of an ideal rigidly plastic medium, the problem under consideration with $l \geq 0.5$ has solutions (Fig. 3) analogous to the solution of Hill and Prandtl for the impression of a single punch [1]. According to these solutions

$$p = 1 + \pi/2. \tag{3.5}$$

The dashed line in Fig. 4 corresponds to an evaluation of (3.4) and the solid line to the value of p according to (3.5). The difference between the evaluation and the value of p according to (3.5) with $0.5 \leq \lambda \leq 2$ does not exceed 12%. The considerable difference with larger values of λ is explained by the fact that, in this case, a continuous plastic zone is not formed in the surface layer of the medium and, consequently, the condition $a_1 + a_2 = l$ corresponds poorly to the actual picture of the deformation. In this case, a better evaluation can be obtained by assuming that, in the region $a_1 + a_2 < x \leq l$, the velocities are equal to zero, and determining a_2 from the condition of a minimum of the evaluation.

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MEASUREMENT OF MULTIPOINT MOMENTS OF COMPOSITE STRUCTURES

V. V. Dudukalenko, N. N. Lysach, and S. I. Meshkov

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Problems of thermal conductivity, elasticity, and the viscosity of multiphase solid materials have solutions in the form of expansions with respect to multipoint moments. However, the use of these solutions is limited by the difficulty in determination of the multipoint moments, giving the random field of the parameters, which are the starting point for a theoretical investigation. Out of the existing quantitative methods of statistical experiment [4, 5], the most promising is optical-structural analysis, based on the storage of a signal, obtained from the output of a pickup of the optical density, with scanning of the structure. Subsequent automated introduction of the recording of the signal into an electronic computer would make it possible to use any given algorithms for calculation of the parameters of the structure.

For justification of the experimental method of determining multipoint moments, let us consider the statistically homogeneous random fields Λ in the space of the Cartesian coordinates X_i. The multipoint moments are determined as a result of averaging

$$\langle \Lambda(X)\Lambda(X')\Lambda(X'')\ldots\rangle = M(X'-X, X''-X, \ldots). \tag{1}$$

For a composite medium, the assignment of the random field of the determining parameters is represented in the form of independent characteristics, relating to the physical properties of each phase separately, and the geometry of the distribution of the phases in space. Let Λ_i be some physical parameter, corresponding

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